COMMON PRE-BOARD EXAMINATION 2023-24

Subject: APPLIED MATHEMATICS (241)

Class XII

MARKING SCHEME

SECTION A

(Each question carries 1 mark)

Q.No.	Hints/ Solution	Marks
1	Option (a) 3	1
2	Option (c) $x \leq 4$	1
3	Option (c) 8.5 km/h	1
4	Option(b) 3:5	1
5	Option (d) 120 seconds	1
6	Option (d) t^2	1
7	Option (c) ₹ 40, 600	1
8	Option (d) any point on the line segment joining the points (0, 2) and (3, 0)	1
9	Option (a) 57	1
10	Option (b) 3	1
11	Option (a) 0.4	1
12	Option (b) accepted	1
13	Option (c) Irregular Trend	1
14	Option (c) 19,25,31	1
15	Option (a) ₹ 8000	1
16	Option (a) 56.5%	1
17	Option (b) ₹ 5000	1
18	Option (b) 6.09%	1

ASSERTION-REASON BASED QUESTIONS

19	Option (c) A is true, but R is false	1
20	Option (a) Both A and R are true and R is the correct explanation of A	1

SECTION B

(Each question carries 2 marks)

21 Given R= 500, P = 10,000 and $i = \frac{r}{200}$

$P = \frac{R}{i}$

$i = \frac{R}{P} = \frac{1}{20}$ 1

$\frac{r}{200} = \frac{1}{20}$

r = 10% per annum 1

22 As the points P (3, -2), Q (8, 8) and R (k, 2) are collinear

Area of triangle PQR $= \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$ ½

Solving above determinant, we get,

3(8-2) +2(8-k) +1(16-8k) = 0 ½

18 + 16 – 2k +16 -8k = 0 ½

-10k + 50 = 0

k = 5 ½

OR

$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$A^2 - 4A + I = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ½

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ ½

Hence proved.

23 Let number of cakes of each kind be x and y ½

Maximize: Z = x + y ½

Subject to constraints:

$2x + y \le 50$ ½

$x + 2y \le 40$ ½

$x, y \ge 0$

24 $3^{50} \bmod 7 = (3^2)^{25} \bmod 7$

$= (2)^{25} \bmod 7$ ½

$= 32^5 \bmod 7$

$= (4)^5 \bmod 7$ ½

$= (16 \times 16 \times 4) \bmod 7$

$= (2 \times 2 \times 4) \bmod 7$ ½

$= 2$ ½

OR

Let the rate at which the stream is flowing be x km/hr and let the distance covered by the boat be d km.

Given, 1

$\frac{3d}{5+x} = \frac{d}{5-x}$ ½

$3(5-x) = 5 + x$ ½

$x = 2.5$

The stream is flowing at the rate of 2.5 km/hr

25 $\frac{dx}{x} + \frac{dy}{y} = 0$

$\frac{dx}{x} = -\frac{dy}{y}$ ½

Integrating both sides,

log x = - log y + log c ½

log xy = log c ½

xy = c ½

SECTION C

(Each question carries 3 marks)

26 $\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$

$\frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$ ½

A = $\frac{1}{2}$, B = - 4, C = $\frac{9}{2}$ 1

$\int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{2(x-1)} + \frac{-4}{(x-2)} + \frac{9}{2(x-3)} dx$ ½

$= \frac{1}{2} log|x-1| - 4log|x-2| + \frac{9}{2}\ log|x-3|$ + C 1

OR

$\int (x^2+1)\ logx\ dx$

Integrating by parts,

$\int (x^2+1)\ logx\ dx = \log x \int (x^2+1)dx - \int ((\log x)' \int (x^2+1)dx)dx$ 1

$= \log x\left(\frac{x^3}{3} + x\right) - \int \frac{1}{x}\left(\frac{x^3}{3} + x\right) dx$ 1

$= \log x\left(\frac{x^3}{3} + x\right) - \int \left(\frac{x^2}{3} + 1\right) dx$

$= \log x\left(\frac{x^3}{3} + x\right) - \left(\frac{x^3}{9} + x\right) + C$ 1

27 3x – 2y + 3z = 8

2x + y – z = 1

4x – 3y + 2z = 4

AX = B

A = $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, X = $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$, B = $\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ ½

$\Delta = |A|$ = $\begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$ = -17 ½

½

$\Delta\, x = \begin{vmatrix} 8 & -2 & 3 \\ 1 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = -17$ ½

$\Delta\, y = \begin{vmatrix} 3 & 8 & 3 \\ 2 & 1 & -1 \\ 4 & 4 & 2 \end{vmatrix} = -34$ ½

$\Delta\, z = \begin{vmatrix} 3 & -2 & 8 \\ 2 & 1 & 1 \\ 4 & -3 & 4 \end{vmatrix} = -51$ ½

$x = \frac{\Delta\, x}{\Delta} = 1$, $y = \frac{\Delta\, y}{\Delta} = 2$, $z = \frac{\Delta\, z}{\Delta} = 3$

28 λ= 3.2

a) $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$ ½
   $P(X = 0) = 0.041$ ½
   $P(X = 1) = 0.13$ ½
   $P(X = 2) = 0.21$ ½
   $P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.041+0.13+0.21 = 0.381$ ½
b) $P(X \ge 3) = 1 - P(X \le 2) = 1 - 0.381 = 0.619$ ½

29 $f(x) = 2x^3 - 9x^2 + 12x + 15$

$f'(x) = 6x^2 - 18x + 12 = 0$

$6(x-2)(x-1) = 0$

$x = 1, 2$ 1

The points divide the real line into three intervals $(-\infty, 1), (1, 2), (2, \infty)$

$f'(x) > 0$ in $(-\infty, 1)$ and $(2, \infty)$.

Therefore, $f(x)$ is strictly increasing in $(-\infty, 1)$ and $(2, \infty)$. 1

$f'(x) < 0$ in $(1, 2)$.

Therefore, $f(x)$ is strictly decreasing in $(1, 2)$. 1

OR

$p = 35 - 2x - x^2$

$p_0 = 20$

$35 - 2x - x^2 = 20$

$x^2 + 2x - 15 = 0$ 1

$(x+5)(x-3) = 0$

$\therefore x_0 = 3$

Consumer's Surplus $= \int_0^{x_0} f(x)dx - p_0\, x_0$ 1

$= \int_0^3 (35 - 2x - x^2)dx - 60$

$= \left[35x - x^2 - \frac{x^3}{3}\right]_0^3 - 60$ 1

$= 105 - 9 - 9 - 60$

$= 27$

30 H_0: μ = 0.50 mm

H_1: $\mu \neq$ 0.50 mm 1

$t = \frac{\bar{X}-\mu}{s}\sqrt{n-1} = \frac{0.53-0.50}{0.03} \times 3 = 3$ 1

Since the calculated value of t i.e. t_{cal}(=3) > t_{tab}(=2.262), the null hypothesis H_0 can be rejected. Hence, we conclude that machine is not working properly. 1

31 Here P = 9,50,000, i = 0.0125

n = 48 1

Using reducing balance method,

$$E = \frac{Pi}{1-(1+i)^{-n}} = \frac{9,5,0000 \times 0\cdot0125}{1-(1+0\cdot0125)^{-48}}$$ 1

$$= \frac{11875}{1-(1.0125)^{-48}} = \frac{11875}{1-0.5508565}$$

$= ₹\ 26,439\cdot21$ 1

SECTION D

(Each question carries 5 marks)

32

$$A = \frac{R[(1+i)^n - 1]}{i}$$ 1

R=10000, n=20, I =10% p.a. 1

$$A = \frac{10000[(1+0.1)^{20}-1]}{0.1}$$

Cost of the machinery = ₹ 573000 1

If n =10 1

$$A = \frac{10000[(1+0.1)^{10}-1]}{0.1}$$

Cost of the machinery = ₹ 159400 1

OR

i) $D = \frac{C-S}{n}$, where C= original cost , S= scrap value , n=useful life

Here C=2,00,000/- , S=10,000/- and n=6 years 2

D=31666.67

Therefore annual depreciating cost for her responsibility is Rs31667(approx.)

ii) 3

YEAR	BOOKVALUE (BEGINNING OF EACH YEAR)	DEPRECIATION	BOOKVALUE (AT THE END OF EACH YEAR)
1	2,00,000	31667	168333
2	168333	31667	136666
3	136666	31667	104999
4	104999	31667	73332
5	73332	31667	41665
6	41665	31667	9998

33 Let x be the number of packages of screws A and y be the number of packages of screws B that the factory manufactures.

Clearly, $x, y \geq 0$.

Total time on machine is 4 hr = 240 min ½

∴ for automatic machine,

$4x+6y \leq 240 \Rightarrow 2x+3y \leq 120$ ½

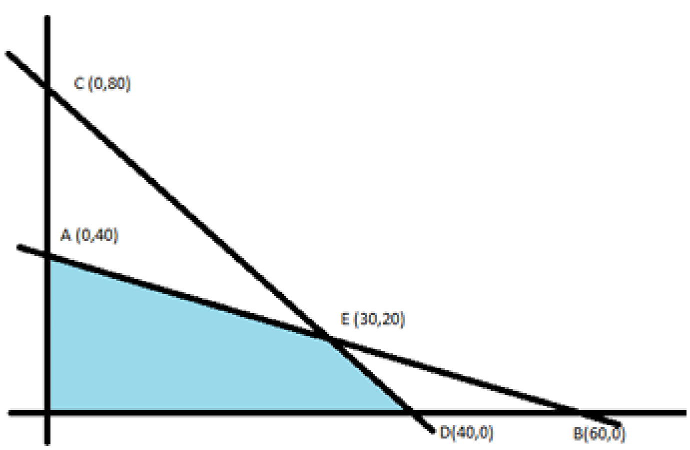
∴ for hand machine,

$6x+3y \leq 240 \Rightarrow 2x+y \leq 80$ ½

The profits on Screw A is ₹ 7 and on Screw B is ₹ 10.

We need to maximize the profits,

i.e. maximize $z = 7x+10y$, given the above constraints. ½



2

The shaded region is the feasible region.

Now,

at A, z=28

at O, z=0

at E, z=41

at D, z=40

Hence, maximum profit is at point E(30,20), i.e at x = 30 and y = 20. 1

34 $C(x) = 100 + 0.025x^2$

$R(x) = 5x$

P(x)= R(x)–C(x)

$= 5x - 100 - 0.025x^2$ 1

P'(x) = 5 – 0.05x

If P'(x) = 0, x = 100 1

P''(x) = - 0.05

∴ Manufacturing 100 dolls will maximize the profit of the company 1

$P(x) = 5x - 100 - 0.025x^2$

When x= 100, P(x) = 500 – 100 -250 = 150 1

Maximum Profit=₹1,50,000 1

OR

Let each side of the square base of tank be 'x' cm and its depth be 'y' cm. 1

Then, V (Volume of the tank) = $x^2y = 4000$

$$y = \frac{4000}{x^2}$$

If 'S' is the surface area of the tank, then 1

$$S = x^2 + 4xy$$

$$= x^2 + 4x\left(\frac{4000}{x^2}\right)$$

$$= x^2 + 4\left(\frac{4000}{x}\right)$$ 1

$$S' = 2x - \frac{16000}{x^2}$$

When S' = 0, x = 20 1

$$S'' = 2 + \frac{32000}{x^3}$$

When x = 20, S'' = 6 > 0

∴ S (The surface area of the tank) is minimum for x = 20cm, y = 10cm 1

35 Let x, y and z denote the quantity of first, second and third product produced respectively.

x + y + z = 45

z = x + 8 1

x + z = 2y

Using Matrix Algebra,

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$ ½

½

AX = B

$|A| = 6$

Adj A = $\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$ 1

½

$A^{-1} = \frac{1}{|A|}$ Adj A = $\frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$X = A^{-1}B$ ½

$$= \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$$ 1

Therefore, the quantity of first, second and third product produced respectively are 11 tons, 15 tons and 19 tons.

SECTION E

(This section comprises of 3 source-based questions (Case Studies) of 4 mark each)

36 Let x,y and z be the time taken by taps A, B and C respectively to drain the tank separately

(i) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{3}$

$\frac{1}{y} + \frac{1}{z} = \frac{1}{2}$

$\frac{1}{x} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ 1

Therefore, tap A takes 6 minutes to drain the tank separately.

(ii) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{3}$

$\frac{1}{x} + \frac{1}{z} = \frac{13}{30}$

$\frac{1}{y} = \frac{2}{3} - \frac{13}{30} = \frac{7}{30}$

Therefore, tap B takes $4\frac{2}{7}$ minutes to drain the tank separately. 1

(iii) a) $\frac{1}{x} + \frac{1}{y} = \frac{1}{6} + \frac{7}{30}$

$= \frac{12}{30} = \frac{2}{5}$

Therefore, tap A and B will together take $2\frac{1}{2}$ minutes to drain the tank separately 2

OR

b) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{3}$

$\frac{1}{z} = \frac{2}{3} - \frac{1}{6} - \frac{7}{30}$

$\frac{1}{z} = \frac{8}{30} = \frac{4}{15}$

Therefore, tap C takes $3\frac{3}{4}$ minutes to drain the tank separately.

2

37 (i) a) 0 + k + 6k + 4k + 2k = 1

k = $\frac{1}{13}$ 2

OR

(i) b) P (X $\leq$ 3) = 0 + k + 6k + 4k = 11k = $\frac{11}{13}$ 2

(ii) P (X = 2) = 6k = $\frac{6}{13}$ 1

(iii) P (X = 4) = 2k = $\frac{2}{13}$ 1

38

Year (x_i)	Index Number(Y)	X = $x_i - A$ = $x_i - 2007$	X^2	XY
2004	18	-3	9	-54
2005	20	-2	4	-40
2006	23	-1	1	-23
2007	25	0	0	0
2008	24	1	1	24
2009	28	2	4	56
2010	30	3	9	90
n = 7	$\sum y$ = 168	$\sum X$ = 0	$\sum X^2$ = 28	$\sum XY$ = 53

2

a = $\frac{\sum y}{n}$ = 24

b = $\frac{\sum XY}{\sum X^2}$ = 1.89 1

Y=24+1.89u ½

For year 2014 we have

Y =24+1.89(2014-2007) =24+1.89 × 7 = 37.23 ½

OR

Year	Index No.	4-year moving total	4-year moving average	Centered total	Centered moving average
1980	400				
1981	470				
		1730	432.5		
1982	450			873	436.5
		1762	440.5		
1983	410			882.25	441.125
		1767	441.75		
1984	432			886.25	443.125
		1778	444.5		
1985	475			911.5	455.75
		1868	467		
1986	461			946	473
		1916	479		
1987	500			946.75	473.375
		1871	467.75		
1988	480				
1989	430				

4